Translated excerpt

Georg von Wallwitz
Meine Herren, dies ist keine Badeanstalt.
Wie ein Mathematiker das 20. Jahrhundert veränderte

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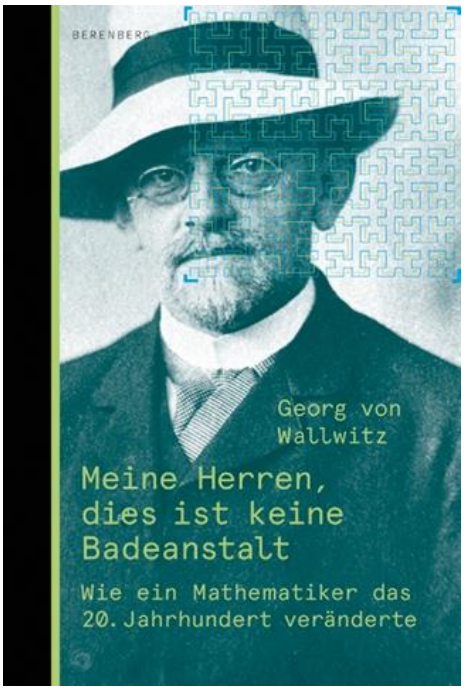
Georg von Wallwitz
Gentlemen, this is not a bathhouse: How a Mathematician Changed the Twentieth Century

Translated by Allison Brown



One often hears: that is good but it
belongs to yesterday. But I say: yesterday has
not yet been born. It has not yet really existed.
Osip Mandelstam, *"The Word and Culture"* (1921)

1. Preface

Basically, there is an amiable breed of people who, when contemplating the world, are willing to go beyond intuition and let themselves be carried by the form and inner logic of the phenomena alone. These people are imaginative and courageous in the face of strict logic and self-confident enough to admit unavoidable defeats after a short period of disappointment—because the truth of formal consideration is more important to them than anything else. They seldom attain the status of a pundit, do not really thirst for glory, and are usually content with a small, quiet room where they create nothing less, as it were, than the lubricant that keeps the modern world rolling. We are speaking, of course, about mathematicians.

We all know the great impact of their work. Through concepts such as *big data, artificial intelligence,* and *cryptography,* mathematical techniques have permeated the daily life of all of humanity. No matter how good someone's schooling is, they often don't have a very clear idea of what modern mathematics actually is and who comes up with things like that. Despite their growing influence, mathematicians have largely lost their connection to major segments of the educated classes.

That was not always the case. In the eighteenth century a philosopher could, like Voltaire, write a book on Newtonian physics, and great mathematicians had a high profile in the salons of Paris, London, and Berlin. Poets such as Novalis thought extensively about Euclidean axiomatics and the significance of binomial formulas. No one would even have considered the notion that an educated person could get by without the most current knowledge of mathematics. By around 1800, however, people were no longer familiar with math as a matter of course. It lost its clarity

through the awe-inspiring abstract work of Carl Friedrich Gauss and some of his contemporaries, and became a subject that by no means could be practiced on the side by cavaliers as an amusing diversion. Dilettantes in mathematics slowly died out. Today it is no longer part of a general education and some extremely cultured people can even admit they haven't the faintest notion of modern math without having to fear social ostracism. From a distance they show interest, taking note of mathematics' claim to be absolute, eternal, and beautiful, but to them the subject seems as accessible as passing through the eye of a needle. It is to these people that I address this book.

Ordinary mathematicians know very well that the general public neither seeks nor grasps the actual substance of their scholarship. As much as they would really like to explain to passersby on the street what topology or algebraic geometry is all about, in the end they will at best bring back vague memories of school years and not really convey any significant concepts. This book therefore cannot discuss the substance; there would be no sense in that. I can only assure readers that it can be a beautiful experience to understand logical relationships and that there continues to be life also in the intellectual world beyond intuition and colorful ideas. One can, of course, talk about the biographical constellations, about the way of thinking and the problems from which mathematics emerged. And then it is possible to be all the more amazed at how a purely intellectual exercise can delve so deeply into reality.

The generation born around 1900 experienced most strongly how deep this intervention has been. In their childhood, state-of-the-art technology was represented by the light bulb and steam engine. Fifty years later there were cars, airplanes, atom bombs, computers, radar, radio, and television. The world was hardly recognizable anymore. Physics brought forth the general theory of relativity and quantum mechanics; and completely new branches of science emerged, such as game theory and cybernetics. In half a century the world had changed more dramatically than in almost any other period in history. As opposed to this storm, the changes that the Western world is experiencing today is at best a slight breeze.

If we were to seek a figure who more often than all the others surfaces at the sources of this upheaval, we would quickly encounter David Hilbert in Göttingen, the most influential mathematician in the first half of the twentieth century. He was the gray eminence behind the scientific minds that shook the world at this time, each in their own way. No one else brought together so many scientists who would later play such a decisive role, and at no other desk did so many connections and ideas cross paths, from which, without having been planned, a new age ultimately emerged. Hilbert set the course for all the mathematical developments of the twentieth century. Much of what we see today in our everyday lives, such as the development of the computer, emerged around discourse on his ideas. And it is not by chance that many of the physicists who later built the atom bomb got to know each other in the 1920s in Hilbert's Göttingen.

"There are, in my view, two factors that, above all others, have shaped human history in the twentieth century. One is the development of the natural sciences and technology, certainly the greatest success story of our time. … The other, without doubt, consists in the great ideological storms that have altered the lives of virtually all mankind,"[1] wrote Isaiah Berlin, one of the best observers of his time. Whereas the external mantle of twentieth-century history consisted of war, destruction, and expulsion, of ideologies, racism, and bigotry, the inside lining was woven out of the phenomenal developments in the mathematical natural sciences, which left just as strong a mark on the shape of the century. The tyrants whom no one could avoid in the twentieth century have been the subject of much writing and discussion—possibly more than they deserved. But in two hundred years people might determine that the ideas and methods conceived in mathematics and physics during the twentieth century changed the course of history more lastingly than the barbarism of the ideologies did.

In a glorious time for mathematics, Hilbert was the outstanding head of a school that offered the natural sciences and technology the means to understand the world in a new way. This school attracted talented young people from around the

[1] Isaiah Berlin, "The Pursuit of the Ideal," in *The Crooked Timber of Humanity*, 2nd ed. (Princeton: Princeton University Press: 2013), 1.

world. They were an eclectic, brilliant bunch, unconventional in every respect. Hilbert stood up and spoke out unconditionally for his students, such as Emmy Noether, whom he had difficulty getting accepted as a lecturer, fighting against his colleagues in the Philosophical Faculty, who remained committed to the archaic image of women during the German Empire. And he only managed to have her teach at all by indicating the difference between a university department and a public bathhouse. His trademark was called the *axiomatic method* in the sense of the great Euclid. It expressed the ambition not only to understand what Faust referred to as the "poodle's core," that is, the heart of the matter, but to put that poodle back together in a logically flawless, formal way. It is an attempt to understand things based on their inner logic. This was an intellectual revolution, a break from a Romantic tradition that regarded the mathematician as committed only to his own intuitive genius.

Mathematical knowledge has the structure of a pyramid. In school, most of us confronted that which is basically a dark science, consisting of the proper application of memorized formulas and which only becomes bearable through the use of pocket calculators. This school math makes up the broad base of the pyramid and is objectively boring. Any professional will confirm this suspicion of amateurs. But, as the pros will then assure in the same breath, mathematics is also interesting and beautiful. It is *interesting* at the point where it meets reality and becomes visually accessible. A large part of mathematics evolved out of concrete problems and it becomes tangible at this interface between intellect and nature: For example, where the homing abilities of Tunisian desert ants can best be understood as operating with vectors. Or where the smartest approach in gambling becomes a subject of probability calculus. And mathematics is *beautiful* at the very tip of the pyramid, where it can become an aesthetic experience, when the arduous advancement in number theory, topology, or algebra is rewarded with a feeling of eternal truth and harmony. There it has a lot to do with inspiration and free play of forms, which since time immemorial have been associated with the sensual experience of beauty. Once the strict framework of devising thematic concepts has been mastered, a totally

different picture emerges. It is as if the tip of the pyramid juts out of a sea of clouds of unfocused and disjointed concepts.

Lay people in math are well-advised not to beat their way through the formula jungle that is raging between them and the good thoughts at the pyramid's tip, but instead to pay attention to the style and the path. How should a lay person read a book about mathematics? In every field there is a particular jargon and practitioners need a long period of training to become familiar with it—like dancers learning their steps. If terms and passages come up in this book that remain unclear, I ask readers to be patient and to have the courage at first to read past the hard parts and keep to the essence. This is not a matter of precise definitions, but of a series of great ideas—some of the most influential ones that the past century had to offer. I have relegated most of whatever goes beyond school mathematics to footnotes marked as "MMM": "math maven material."

What might appear to be a cursory discussion is definitely not insulting to mathematicians. When reading a treatment, they too like to skip over passages that seem difficult. They usually first read the sentences that represent the distillate of the considerations. Although they know that often only the proof clearly expresses the meaning of a sentence, at times they nevertheless only go through the steps in the proof if they have the feeling that a good idea lies behind it. Not every mathematician is diligent and willing to deal with the difficult stuff. And their readers don't have to be either.

Pay attention to the patter by means of which we convince someone of the truth of a mathematical proposition. It tells us something about the function of this conviction. I mean the patter by which intuition is awakened.

Ludwig Wittgenstein[2]

3. Two Birds, a Frog, and the Archangel of Progress

Königsberg, where Hilbert grew up in the second half of the nineteenth century, was a well-ordered, proud, and open city, East Prussia's port to the world and coronation city in what was then the most modern country in Europe. Nevertheless, the city's best days were long past. The port and the stock exchange were large and significant, and Königsberg could boast of being the world's major trade center for peas and Germany's largest port for grains and wood.[3] But these commodities had lost much of their status in the last third of the nineteenth century. Liverpool and New York were the ports of the future, where peas played a more secondary role. Steel, clothing, and machinery were produced elsewhere, and East Prussia was only able to watch these developments from afar. The Königsberg hinterland had fallen behind socially, economically, and culturally, dominated by puritanical landowning families that viewed the new technologies and capitalist economies as a threatening bad weather front rising up in the western part of Germany. And so the city remained noteworthy in the nineteenth century, but only in a region at the margins of the

[2] Ludwig Wittgenstein, *Remarks on the Foundations of Mathematics*, ed. G.H. von Wright, R. Rhees, and G.E.M. Anscombe, trans. G.E.M. Anscombe, rev. ed., part 4, §27 (Oxford: Blackwell and and Cambridge, MA: MIT Press, 1978), 237.
[3] See Fritz Gause, *Die Geschichte der Stadt Königsberg in Preussen*, vol. 2 (Cologne: Böhlau, 1996), 668ff.

empire that continued to lose commercial and cultural weight. The fortifications, both the physical and the intellectual ones, were great (due to the proximity to Russia and the pride of the kings), but unkempt.

Foreign visitors quickly felt the relative stagnation. Königsberg (as well as its university, the Albertina) continued to live off the legends surrounding Immanuel Kant; in its liberalness and punctuality it was a real-life expression of the intellectual world of its greatest son, a "city of pure reason and dirty streets."[4] In general, the Königsberg professors were good, but they preferred to spend their time in Berlin or even farther away, at the pulse of a world that was changing ever faster in the West. The Albertina was the fourth smallest of Prussia's twenty universities and into the 1870s it had only a little more than three hundred students. The person and work of Kant gave it an extraordinary heritage, a spirit that hovered over all of the academic departments and had to be mentioned in every lecture. But three generations after the death of the philosopher this constant reference to one and the same intellectual anchor no longer seemed particularly novel.

The state of the Mathematics Seminar fit well into this neglected research landscape. It had no facilities of its own and was located within the university's own student prison. The library consisted essentially of the volumes of the *Mathematische Annalen*. There were no blackboards in the lecture hall that were large enough to fit longer formulas or proofs—which, while encouraging trenchant brevity, did not always do justice to the subject at hand. Allocation of the lecture hall did not follow any set schedule, but was based on the seniority of the professors wanting to teach there, as if it were assumed that it was interesting to hear what old men had to say. In this situation it was virtually inevitable that true mathematics would be taught outside of the university—out of doors if the weather was good, and in pubs when it rained.

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In the summer of 1884, three young men met at 5 o'clock almost every afternoon at Paradeplatz, the square in front of the main building of the Albertina, to stroll in the

[4] Felix Dahn, *Erinnerungen*, vol. 4 (Leipzig: Breitkopf & Härtel, 1890–95), 69f; cited in Jürgen Manthey, *Königsberg* (Munich: Hanser, 2005), 533.

sun and traverse the mathematical and scientific knowledge of their day. The men were Adolf Hurwitz, a *Privatdozent*, or adjunct professor, and two doctoral candidates, Hermann Minkowski and David Hilbert. For Hurwitz, the youngest and lowest-ranking *Privatdozent* at the Mathematics Seminar, it was next to impossible to be allocated a lecture hall, and so it made perfect sense for him to seek out the students who were most eager to walk and drink with him for his instruction outside the university building. At first he found only two, but as regards a revolution of ideas, the number of fires the insurgents set is in any case less important than the state of exhaustion of the old regime.

They were three gaunt figures who wore mighty moustaches and close-fitting suits of a heavy fabric, very much in keeping with the current fashion. They were always deeply engaged in their conversation, and the level of concentration with which each of them listened to the others, to be sure not to miss even a nuance of what was said, gave the impression they were walking under a bell jar that largely isolated them from the world around them. Their walks led them through the King's Garden to the castle pond, where the student fraternities and the rowing clubs had their quarters. At that time the pond was a little over a kilometer long, stretching from the city center to the northern fortifications. For a while already, it was possible and even common for younger ladies to go rowing there in the afternoon sun, offering with their white blouses and wide-brimmed hats an unmistakable eye-catcher for flaneurs. However, these three were by no means carefree idlers, and during the most intense and important hour of their day, they had no eyes for these women. They strolled through the Börsengarten without glancing either left or right, past the restaurant with the same name, built in the style of an open-air Bavarian beer garden, which could be considered the greatest everyday pleasure in this punctual Protestant city. The promenade around the lake was lined with numerous park benches that offered the ideal conditions for relaxing, dozing, chatting. These three men, however, would not even consider sitting on a bench like members of the complacent property-owning bourgeoisie. Their steps provided the cadence and grounding for their thoughts, which had to be constantly expounded and verified, so that the men would not lose their orientation and their cohesion as if an autumnal

fog coming from the Baltic Sea had suddenly settled over them during their walk. Their gait, the movement, and the ritual were all essential parts of the conversation, the physical reflection of their intellectual headway.

Their walks proceeded along the lakefront to the north, past the prominent houses of the "Three Crowns," "To the Skull and Phoenix," and "Immanuel" freemason lodges; past the Wilhelms-Gymnasium secondary school, the Baptist church, and finally also the Dohna Tower, which marked the point where the newer fortifications passed between the castle pond and the Upper Lake (Obersee). Here the strollers left the Old Town behind them, walking through the parks that had been established where the medieval city walls once stood, until they finally reached an apple tree, the destination of their daily walk.[5] Then it was time to pause and identify the initial results of their conversation, the content of which was, after all, not easy to digest. This was because, in the end, mathematical objects are only to a certain degree suitable for beer halls and strolls. They require intense concentration; nothing can be omitted; everything must be carefully and properly derived and combined; nothing can be kept in a quivery, approximate state; neither an unfounded beginning nor an open end is acceptable. A calculation is not a calculation, and a proof not a proof if there are any unexplained gaps or if inadmissible auxiliary aids appear out of nowhere. Mathematicians can be terribly pedantic and lacking a sense of humor, which is why at some point it makes sense to move to a closed and quiet room to work it all out on paper, so that the train of thought remains traceable and in proper order. Retaining the argument of long calculations or multiple proof steps in one's head, while at the same time working further on it, can overwhelm the brain. Even the most talented inevitably reach the limits of human mental capacity. At this point at the latest, the stroll or the gathering in the restaurant was to be brought to an end in order to verify whether or not the ideas were truly as brilliant as they just seemed.

[5] The walks could be reconstructed with the help of a contemporary city map and Hilbert's obituaries for Hurwitz and Minkowski, as well as the following sources: Minkowski's letter to Hilbert of April 16, 1895, in Constance Reid, *Hilbert* (Berlin: Springer, 1970), 122; Ferdinand von Lindemann, *Lebenserinnerungen* (Munich: self-published, 1971), 90ff; and David Rowe, "From Königsberg to Göttingen: A Sketch of Hilbert's Early Career," *The Mathematical Intelligencer* 25, no. 2 (2003): 44–50.

Hurwitz, only slightly older than the two students, had unruly hair that stood up like bristles, with the texture and length of those on a shoe brush. His moustache hung down grimly in walrus style, like one we would today associate only with photographs of Nietzsche. The overall impression he gave was not a healthy one, after having contracted typhus while a student at the Technical University in Munich. He often suffered from migraines, looked slight, delicate, and frail, but he had lively, happy eyes. His appearance was inconspicuous and "nothing was farther from Hurwitz than to appear Bohemian or eccentric. He was always correct, reserved, inconspicuous, exceedingly modest, lifting his hat to the servants of the neighbors. A stranger could not suspect that there was more behind this unassuming exterior than middle class respectability."[6] And only someone with a sense of mathematics could ever see, in this somewhat sickly and delicate figure, any confirmation of his reputation as a child prodigy. He was an extremely talented musician as well.

Hermann Minkowski, the youngest of the three who took their walks together, was even more of a wunderkind than Hurwitz. Endowed with boundless talent, he passed his university qualification examination at fifteen, and at seventeen had his first major international appearance, when in 1881 he solved the prize question (regarding the number of representations of an integer as the sum of five squares) of the Paris Academy of Sciences in the competition for the Grand Prix des Sciences Mathématiques. This episode is rather amusing since the problem had already been solved by Henry Smith, a professor of average talent at Oxford, without those in Paris having taken note of it. (It was not unusual that this achievement in Oxford would slip through without the notice of the Paris Academy, because scientists of this era avoided following the publications of another country unless it was absolutely necessary.) This was significant because of how it highlighted the young Minkowski. He solved the problem with such brilliance that the Academy really wanted to award the prize to him, despite the objections raised by nationally inclined French and British. But Charles Hermite and Camille Jordan, who at the

[6] George Pólya, "Some Mathematicians I Have Known," *American Mathematical Monthly* 76, no. 7 (Sept. 1969): 746–753, here: 751.

time were the authoritative heads of their subject in Paris, were able to distinguish Smith's unsophisticated entry from Minkowski's genius, and they stuck to their decision. Jordan recognized the talent of the adolescent author like an artist sees the sculpture hidden in a block of marble, and wrote to him: "Travaillez, je vous prie, à devenir un géomètre eminent." ("Work, I pray you, to become a great mathematician.")[7] The request was answered.

The Minkowski family had only immigrated a few years earlier from Aleksotas (which now belongs to Kaunas in Lithuania). They no longer felt at ease under czarist rule, ever since the Polish-Lithuanian territories started being oppressed, silenced, and taxed in the aftermath of the Uprising of 1863. The better educational opportunities and the size of the Polish community in nearby Königsberg also contributed to their decision to relocate.[8] Minkowski's older brother Maxim had already gone to an academic secondary school (*Gymnasium*) in Insterburg, near Königsberg.

Despite his obviously extraordinary talent, Hermann Minkowski has been described as a very modest and shy person who tended to stutter, as if his talent were embarrassing to him. What can a boy do with such a powerful sense of reason, who boiled his school years down to months and turned into child's play that which for others was the culmination of years of arduous efforts? He read books on mathematics and the natural sciences, and to relax, Shakespeare's *Othello* and Goethe's *Faust*. His humor, which kept flashing in his later letters, was of the sort cultivated from the reserved observer role that Minkowski assumed in social settings, half sought and half forced upon him. In any case he later regretted never having had time in his youth to be reckless and carefree.

"No mathematician should ever allow himself to forget that mathematics, more than any other art or science, is a young man's game." It is not a place where old people can still move anything, and "I do not know an instance of a major

[7] Cited in David Hilbert, *Minkowski,* in Hilbert, *Gesammelte Abhandlungen* (*GA*, Collected Papers), vol. 3 (Berlin: Springer, 1935), 341. English translation cited in Constance Reid, *Hilbert*, 12.
[8] See Lily Rüdenberg, "Erinnerungen an H. Minkowski," introduction to Hermann Minkowski, *Briefe an David Hilbert* (Berlin: Springer, 1973), 12.

mathematical advance initiated by a man past fifty,"[9] wrote G. H. Hardy, the most eccentric of the outstanding mathematicians of the twentieth century. Young people are not yet obstructed by the certainties of age, are not yet vain and saturated with experiences, have no commitments to methods and schools, have all the freedom to err and to take less respectable detours. They are naïve enough to try out even that which is terribly simple, which is sometimes the solution when the complexity of a problem threatens to burst our brains. They are not so entangled as their teachers in the techniques that the teachers grew up with and can blithely try something new. "Every mathematical soldier carries a marshal's baton in his knapsack, if he does not swear by everything given, out of pure discipline,"[10] Minkowski later noted. In contrast to the historical sciences, in which one scholarship usually stifles another, mathematics has not become less accessible through its progress. "Although mathematics today represents such a mighty and expansive structure, the entrances are becoming increasingly open, the rooms always lighter and more transparent, and if only you forge the right key to the gate, you can immediately move into the innermost depths."[11] With these words, Minkowski was probably thinking not least of his own heroic deeds in his youth.

In this sense, David Hilbert—the second doctoral candidate that Hurwitz attended to on his strolls—was not a promising talent. There are no marvelous things from his school days to report, at most problems with the ancient languages and good grades in arithmetic. He completed secondary school without any major highs

[9] Hardy, *A Mathematician's Apology,* sec. 4 (Cambridge: Cambridge University Press, 1967), 70, 72. Norbert Wiener also offered the reason many mathematicians become depressed once they reach fifty: "Mathematics is very largely a young man's game. It is the athleticism of the intellect, making demands which can be satisfied to the full only when there is youth and strength. After one or two promising papers, many young mathematicians who have shown signs of ability sink into that very same limbo which surrounds yesterday's sports heroes. … Yet it is not bearable to contemplate a brief distinction and burgeoning of activity which is to be followed by a lifetime of boredom." Norbert Wiener, *I Am a Mathematician* (Garden City, NY: Doubleday and Cambridge, MA: MIT Press, 1956), 42.
[10] Hermann Minkowski, *Peter Gustav Lejeune Dirichlet,* in *Gesammelte Abhandlungen*, vol. 2 (Leipzig: Teubner, 1911), 447–461, here: 459.
[11] Letter to Hilbert of July 17, 1902, in Hermann Minkowski, *Briefe an David Hilbert* (Berlin: Springer, 1973), 150.

or lows and with no great enthusiasm for any particular subject, so that he later felt compelled to apologize that "I wasn't all that concerned with math during my school days, because I knew I would do that later."[12] He was—as was said back then in East Prussia—"*dammelig*,"[13] a bit dull and silly. He was twenty-two at the time (two and a half years older than Minkowski, who, however, completed his *Abitur* exam to gain university qualification a semester earlier) and already had the appearance of a certified accountant, with thin hair, rather protruding ears, a pince-nez on his nose, and a pointed chin.

The Hilbert family personified the Protestant spirit—described by Kant as stiff, punctual, and honest, though nevertheless eager to take risks—which was displaying its final splendor in Königsberg. Their life was well-ordered, the week was for working, Sundays for church, and the summer vacations were spent at the nearby Baltic Sea. Hilbert's great grandfather liked to hike as a boy, working his way up from being a barber in Freiberg, Saxony, to an army doctor in the Seven Years' War, finally becoming the "city physician, surgeon, and accoucheur"[14] in Königsberg. The extended family's sons were often named David, which was an external sign of an internally fading pietistic heritage. Their wives—daughters of schoolmasters—brought more upbringing than education to the family, and gave their children more of a sense of duty than culture, in order to swim along in a society whose steadily growing middle class left doubting to the philosophers and dreams of a better world to the socialists, communists, and anarchists. The father of the Hilbert who went on walks with Hurwitz and Minkowski was a local court judge like his father, bourgeois, strict, "a somewhat one-sided jurist so connected with Königsberg and set in his regular habits that he took the same walk every day."[15] His mother was a

[12] Cited in Otto Blumenthal, *Lebensgeschichte,* in Hilbert, *GA,* vol. 3, 389.
[13] Constance Reid, *Hilbert*, 5. Reid's book is the only comprehensive biography of Hilbert. She bases her work mainly on statements by contemporaries and is therefore the most important source for all biographical events. The most significant scholarly source on Hilbert is a series of articles by David Rowe in various publications.

[14] Otto Blumenthal, *Lebensgeschichte,* in Hilbert, *GA,* vol. 3, 388. An accoucheur is a male midwife.
[15] Ibid., 389.

respectable wife of a local court judge. She came from a mercantile family and in quiet hours also busied herself with astronomy and computing prime numbers. Otherwise there was nothing significant to report about her, nothing about special interests, whether in art or music or politics. It was all so common and typical of the educated middle class, so well-ordered, so heartily gray, that it was virtually a miracle that the youngest son, David Hilbert, did not follow the wishes of his father and also become a Prussian public servant.

That was the configuration that became the fertile ground for some of the best ideas of the twentieth century. The unremarkable Hilbert and Minkowski, who was extraordinary very early on, were taken under the wings of Hurwitz and led through the vast world of mathematics. The two wunderkinder recognized in Hilbert not only a *dammelige* boy, but also a profound, slowly maturing talent. "That I have been able to accomplish anything in mathematics," Hilbert said later, "is really due to the fact that I have always found it so difficult. When I read, or when I am told about something, it nearly always seems so difficult, and practically impossible to understand, and then I cannot help wondering if it might not be simpler. And … on several occasions it has turned out that it really was more simple!"[16] Hurwitz and Minkowski recognized Hilbert's depth in his slowness and the originality in his naïve way of doing things. In fact, as would soon become apparent, he was blessed with enough talent to keep pace with the others on the five o'clock strolls to the apple tree.

"Some mathematicians are birds, others are frogs. Birds fly high in the air and survey broad vistas of mathematics out to the far horizon. They delight in concepts that unify our thinking and bring together diverse problems from different parts of the landscape. Frogs live in the mud below and see only the flowers that grow nearby. They delight in the details of particular objects, and they solve problems one at a

[16] Cited in Constance Reid, *Hilbert*, 168.

time."[17] That's how Freeman Dyson classified his guild. Based on this characterization, Hurwitz was a frog and Minkowski and Hilbert were birds.

[17] Freeman Dyson, *Birds and Frogs: Selected Papers of Freeman Dyson, 1990–2014* (Hackensack, NJ and London: World Scientific, 2015), 137.